# CAMBRIDGE INTERNATIONAL EXAMINATIONS General Certificate of Education Ordinary Level 

## MATHEMATICS (SYLLABUS D)

## Paper 2

October/November 2003

## 2 hours 30 minutes

Additional Materials: Answer Booklet/Paper<br>Electronic calculator Geometrical instruments Graph paper (3 sheets) Mathematical tables (optional)

## READ THESE INSTRUCTIONS FIRST

Write your answers and working on the separate Answer Booklet/Paper provided.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paperclips, highlighters, glue or correction fluid.

## SECTION A

Answer all questions.

## SECTION B

Answer any four questions.
At the end of the examination fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
Show all your working on the same page as the rest of the answer.
Omission of essential working will result in loss of marks.
The total of the marks for this paper is 100.
You are expected to use an electronic calculator to evaluate explicit numerical expressions. You may use mathematical tables as well if necessary.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For $\pi$, use either your calculator value or 3.142 , unless the question requires the answer in terms of $\pi$.
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## Section A [52 marks]

Answer all the questions in this section.
1 (a) John opened a bank account.
He deposited $\$ 800$ in his account.
The account pays simple interest at the rate of 5\% per year.
Calculate the total amount in his account after 3 years.
(b) Two telephone companies have different ways of charging their customers.
(i) Michael uses Company A which charges 6 cents for each unit or part unit of time.

A unit of time is 200 seconds.
He makes a call lasting 1 hour 22 minutes.
Calculate the cost of his call.
(ii) Norman uses Company B which charges 5 cents for each of the first 400 units.

The charge for each additional unit is reduced by one quarter.
There is also a fixed charge of $\$ 27$ for the use of the equipment.
He is charged for 629 units.
Calculate, correct to the nearest cent, the total sum that he has to pay.

2 A company manufactures biscuits.
(a) One batch of biscuits contains 300 grams of dried fruit.

This consists of sultanas and currants, with masses in the ratio $2: 3$.
Find the mass of the sultanas.
(b) The mixture used to make one batch of biscuits has a mass of 18 kg .

The mixture loses $12 \%$ of its mass when it is cooked to make the biscuits.
(i) Calculate the mass of one batch of biscuits.
(ii) Each biscuit has a mass of 12 grams.

One batch of biscuits is put into packets.
Each packet contains 16 biscuits.
Find how many packets can be filled, and the number of biscuits remaining.
(iii) The total mass of each packet, including packaging, is 201 grams.

Express the mass of the packaging as a percentage of the total mass of a packet.
(c) A trader sells one packet of biscuits for 80 cents.

He makes a profit of $25 \%$ of his cost price.
Calculate the price he paid for a packet of biscuits.

3 (a) Solve the equation $(2 x-3)(x-4)=18$.
(b) A formula used in connection with a mirror is $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$.
(i) Given that $v=9$ and $f=5$, find $u$.
(ii) Express $v$ in terms of $u$ and $f$.
(c) A man bought $a$ eggs at $r$ cents per dozen.

He sold them for $s$ cents each.
Find an expression, in terms of $a, r$ and $s$, for the profit, in cents, that he made.

## 4 Answer the whole of this question on a sheet of graph paper.

The speeds of 50 cars being driven along a stretch of road were recorded.
The table below shows the distribution of the speeds of the cars.

| Speed <br> $(v \mathrm{~km} / \mathrm{h})$ | $20<v \leqslant 40$ | $40<v \leqslant 50$ | $50<v \leqslant 55$ | $55<v \leqslant 60$ | $60<v \leqslant 70$ | $70<v \leqslant 110$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 4 | 14 | 10 | 8 | 10 | 4 |

(a) Using a scale of 1 cm to represent $10 \mathrm{~km} / \mathrm{h}$, draw a horizontal axis for speeds up to $110 \mathrm{~km} / \mathrm{h}$. Using a scale of 4 cm to represent 1 unit, draw a vertical axis for frequency densities from 0 to 2 units.

On your axes, draw a histogram to represent the information in the table.
(b) Write down the modal class of the distribution.
(c) In which interval is the upper quartile of the distribution?
(d) Find the probability that one car, selected at random, had a speed of
(i) less than $20 \mathrm{~km} / \mathrm{h}$,
(ii) more than $60 \mathrm{~km} / \mathrm{h}$.
(e) There is a speed limit of $60 \mathrm{~km} / \mathrm{h}$ on this stretch of road.

Two cars were selected at random.
Calculate the probability that one car was breaking the speed limit and the other was not breaking the limit.


Diagram I
The points $A, B, C$ and $D$ lie on a circle as shown on Diagram I.
$A C$ cuts $B D$ at $P$.
$A D$ is parallel to $B C$.
(a) Show that triangle $B P C$ is an isosceles triangle.
(b) Given that angle $A C B=32^{\circ}$ and angle $D A B=118^{\circ}$, calculate angle $A C D$.
(c)


Diagram II
Diagram II shows the circle in Diagram I and a second circle, centre $O$.
The two circles intersect at $C$ and $D$.
$A D$ produced cuts the second circle at $F$.
$B D$ produced cuts the second circle at $E$.
Angle $D E F=110^{\circ}$.
Calculate
(i) angle $A C E$,
(ii) angle $C O D$.


Diagram I


Diagram II
$A B C D$ is a rectangle in which $A B=8 \mathrm{~cm}$ and $B C=6 \mathrm{~cm}$.
A circular piece of wire, centre $O$, passes through the vertices of the rectangle as shown in Diagram I.
(a) Show that the radius of the circular wire is 5 cm .
(b) Show that angle $A O B=106.3^{\circ}$, correct to 1 decimal place.
(c) Calculate the area of the shaded segment.
(d) The circular wire is cut at $A, B, C$ and $D$, and the four pieces joined to form the shape shown in Diagram II.

Calculate the area enclosed by the wires in Diagram II.

## Section B [48 marks]

Answer four questions in this section.
Each question in this section carries 12 marks.

7 [The volume of a pyramid $=\frac{1}{3} \times$ base area $\times$ height.]


The diagram shows a solid traffic bollard.
It consists of a square-based pyramid, $V A B C D$, attached to a cuboid, $A B C D P Q R S$.
The vertical line, $V N M$, passes through the centres, $N$ and $M$, of the horizontal squares $A B C D$ and PQRS.
$A B=B C=60 \mathrm{~cm}$ and $V N=40 \mathrm{~cm}$.
(a) Calculate
(i) $V A$,
(ii) angle $V A N$,
(iii) angle $V A P$.
(b) Given also that $A P=B Q=C R=D S=80 \mathrm{~cm}$, calculate
(i) the volume of the bollard,
(ii) the total surface area of the sides and top of the bollard.
(c) The highway authority needs to paint the sides and tops of 17 of these bollards.

The paint is supplied in tins, each of which contains enough paint to cover $8 \mathrm{~m}^{2}$.
Find the number of tins of paint needed.

8 A polar explorer is planning an expedition. He investigates three possible routes.
(a) If he travels on route A , which is 800 km long, he expects to cover $x \mathrm{~km}$ per day.

Route B, which is the same distance as route A, has more difficult ice conditions and he would only expect to cover $(x-5) \mathrm{km}$ per day.

Route C, which is 100 km longer than route A, has easier conditions and he would expect to cover $(x+5) \mathrm{km}$ per day.

Write down an expression, in terms of $x$, for the number of days that he expects to take on
(i) route A ,
(ii) route B ,
(iii) route C .
(b) He estimates that route C will take 20 days less than route B .

Form an equation in $x$, and show that it reduces to $x^{2}+5 x-450=0$.
(c) Solve the equation $x^{2}+5 x-450=0$, giving both answers correct to 1 decimal place.
(d) Calculate the number of days that he expects to take on route A .

## 9 Answer the whole of this question on a sheet of graph paper.

An open rectangular tank has a square base of side $x$ metres.
The volume of the tank is $36 \mathrm{~m}^{3}$.
(a) (i) Find an expression, in terms of $x$, for the height of the tank.
(ii) Hence show that the total external surface area of the tank, $A$ square metres, is given by

$$
\begin{equation*}
A=x^{2}+\frac{144}{x} . \tag{1}
\end{equation*}
$$

(b) The table below shows some values of $x$ and the corresponding values of $A$, correct to 1 decimal place, where $A=x^{2}+\frac{144}{x}$.

| $x$ | 2 | 2.5 | 3 | 4 | 5 | 6 | 7 | 8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $A$ | 76.0 | 63.9 | 57.0 | 52.0 | 53.8 | 60.0 | 69.6 | $p$ |

(i) Find the value of $p$.
(ii) Using a scale of 2 cm to 1 metre, draw a horizontal $x$-axis for $2 \leqslant x \leqslant 8$.

Using a scale of 2 cm to $10 \mathrm{~m}^{2}$, draw a vertical $A$-axis for $40 \leqslant A \leqslant 90$.
On your axes, plot the points given in the table and join them with a smooth curve.
(iii) Use your graph to find
(a) the values of $x$ for which the surface area is $65 \mathrm{~m}^{2}$,
(b) the gradient of the curve at $x=6$,
(c) the dimensions of the tank which has the least possible surface area.


An aircraft waiting to land is flying around a triangular circuit $A B C$.
$A, B$ and $C$ are vertically above three beacons, $X, Y$ and $Z$.
$T$ is the control tower at the airport, and $T, X, Y$ and $Z$ lie in a horizontal plane.
$B C=18 \mathrm{~km}, C A=22 \mathrm{~km}$ and $A B=24 \mathrm{~km}$.
(a) (i) The plane is flying at $200 \mathrm{~km} / \mathrm{h}$.

Calculate the time, in minutes and seconds, that the aircraft takes to complete one circuit.
(ii) Calculate the largest angle of triangle $A B C$.
(b) $Z$ is due West of $T$.

The bearing of $X$ from $Z$ is $042^{\circ}$ and the bearing of $X$ from $T$ is $338^{\circ}$.
(i) Find the angles of triangle $T X Z$.
(ii) Calculate $T X$.
(c) The aircraft is flying at a constant height of 2600 metres.

Calculate the angle of depression of the tower, $T$, from the aircraft when it is at $A$.

## 11 Answer the whole of this question on a sheet of graph paper.



The diagram shows triangle $A$ and the straight line $y=x+4$.
Triangle $A$ has vertices $(3,2),(3,4)$ and $(4,4)$.
(a) Using a scale of 1 cm to represent 1 unit on each axis, draw, on a sheet of graph paper, axes for values of $x$ and $y$ in the ranges $-6 \leqslant x \leqslant 6$ and $-6 \leqslant y \leqslant 10$.

Draw and label triangle $A$.
Draw the straight line $y=x+4$.
(b) The transformation M is a reflection in the line $y=x+4$.

The transformation M maps triangle $A$ onto triangle $B$, so that $\mathrm{M}(A)=B$.
Draw and label triangle $B$.
(c) Triangle $C$ has vertices $(-1,2),(1,2)$ and $(1,1)$.

The rotation R maps triangle $A$ onto triangle $C$, so that $\mathrm{R}(A)=C$.
Find
(i) the angle and direction of this rotation,
(ii) the coordinates of the centre of this rotation.
(d) Given that $\operatorname{MR}(A)=D$, draw and label triangle $D$.
(e) The matrix $\left(\begin{array}{rr}1 & 0 \\ -1 & 1\end{array}\right)$ represents the transformation N which maps triangle $A$ onto triangle $E$.
(i) Find the coordinates of the vertices of triangle $E$.
(ii) Describe fully the transformation N .
(iii) Write down the matrix that represents the transformation which maps triangle $E$ onto triangle $A$.

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